# Technical Note 3:

# Photon counting Statistics

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September 2010

A full derivation of $\mathrm{SNR}_{pc}$, the SNR in a photon-counting detector is presented. This differs from that of an ideal detector due to the effects of coincidence losses.

Let $N$ be the mean illumination in photo-electrons per pixel and $n$ the mean photon-counted signal per pixel, i.e. the fraction of pixels that contain one or more photo-electrons. Poisson statistics tells us that

$$n = 1 - e^{-N}, \tag{1}$$

therefore:

$$N = -\ln(1 - n). \tag{2}$$

The noise in a photon counted frame can be derived straightforwardly by considering that only two pixel values are possible: 0 and 1. Pixels containing 0 will have a variance of $N$, those containing 1 will have a variance of $N - 1$. Knowing the fraction of pixels containing each of these two values then allows us to combine the variances in quadrature to yield $\sigma_{pc}$, the rms noise:

$$\sigma_{pc} = \sqrt{[e^{-N}N^2 + (1 - e^{-N})(N - 1)^2]}, \tag{3}$$

$$= \sqrt{(e^{-N} - e^{-2N})}. \tag{4}$$

The photon-counted images must then be processed to remove the effects of coincidence losses. This is done after the component frames within each temporal bin have been averaged to yield a mean value for $n$ for each pixel. The original mean signal $N$ prior to coincidence losses is then recovered by using Equation 2. Although coincidence loss tends to produce a saturation and a smoothing of the image structure, the overall effect is to add a great deal of noise to the observation and for this reason we must avoid a photon-counting detector entering the coincidence loss regime. The amount of extra noise generated can be calculated by considering the change $dN$ in $N$ produced by a small change $dn$ in $n$. The noise in the final coincidence-corrected pixel will then be equal to that in the unprocessed average pixel multiplied by $dN/dn$. From Equation 2 we get

$$N + dN = -\ln[1 - (n + dn)]. \tag{5}$$

This standard differential is then solved to yield

$$\frac{dN}{dn} = (1 - n)^{-1}. \tag{6}$$

Substituting $N$ for $n$ using Equation 1 we get

$$\frac{dN}{dn} = e^N. \tag{7}$$

We then multiply the uncorrected noise given in Equation 4 by this factor to yield the noise in the final coincidence-loss-corrected PC image. $\mathrm{SNR}_{pc}$ is then given by:

$$\mathrm{SNR}_{pc} = \frac{N}{\sqrt{e^N - 1}}. \tag{8}$$

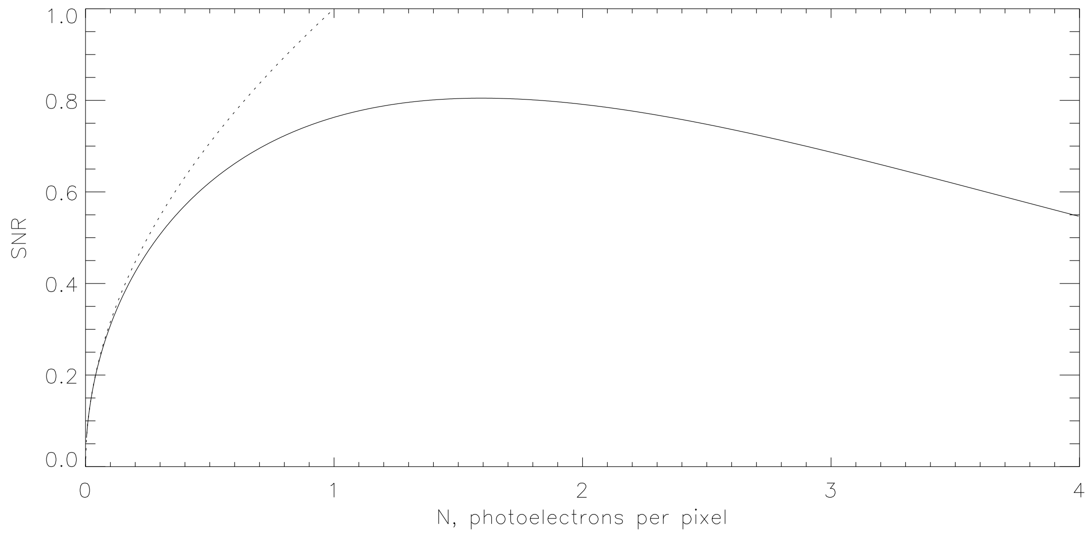


Figure 1: The SNR obtainable with an ideal photon counter. The dotted line shows the SNR obtained with an ideal detector i.e. one that suffers no coincidence losses.

This can be expressed in units of $n$, using Equation 2, which is more useful since it is $n$ that we actually measure from our images:

$$\mathrm{SNR}_{pc} = \frac{-\ln(1-n)}{\sqrt{(1-n)^{-1}-1}}. \tag{9}$$

This function is shown in Figure 1.